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Nuclear Structure and Electromagnetic Transitions of ^{152}Gd nucleus Within Interacting Boson Model (IBM) and Dynamic Deformation Model (DDM)

Saad N. Abood[1], Laith A. Najim[2*] and Enas M. Younis[2]

[1]Physics Department, College of Science, AL-Nahrain University, Baghdad Iraq
[2]Physics Department, College of Science, Mosul University, Mosul Iraq

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Abstract

Collective Models IBM-2 and DDM are employed for a detailed study of the nucleus. Comparison with available experiment data, Energy levels, Electric Transitions Probability B(E2) values, mixing ratio , monopole matrix element and values for some transitions are studied.

Corresponding author; Laith A. Najim: Physics Department, College of Science, Mosul University, Mosul Iraq.
E- mail: drlaithahmed@yahoo.com

Introduction

The earliest microscopic models for this region [1,2] were limited to a static treatment of the deformed energy surface and hence were unable to explain the complex spectrum of a transitional nucleus like ^{152}Gd . Dynamic effects were included in the sixth-boson expansion model and detailed calculations for the N=88 Sm-Er isotones were reported [3]. This was a considerable improvement over the earlier models, but large deviations from experiment were found for the B(E2) branching ratios of many transitions. Another model employed for many nuclei of this region is the interacting boson model (IBM). For instance, Scholten *et al.,* [4] have reproduced the general features of the low-lying energy spectra of the Sm isotopes. But the IBM remains largely a phenomenological model at present and only relative values can be obtained. Similar results for Gd isotopes have recently been obtained by Lipas *et al.,* [5] in a project model where states of good angular momentum are projected from the solutions of a β -vibrator Hamiltonian. Kumar et al., [6] studied collective bands in ^{154}Gd using IBM-1 It is found that the calculated energy values, B(E2) values and interband B(E2) ratios of ^{154}Gd isotope have reasonable agreement with the experimental energies, B(E2) values and B(E2) ratios.

The dynamic deformation model (DDM) [6] is a microscopic model which includes the dynamics of the five-dimensional quadrupole motion and of the pair fluctuations, and which does not require the assumption of small deviations from harmonic vibrations. It has been shown previously that this model is useful for understanding not only the first few collective bands but as many as eight bands. The extremely delicate balance between the spherifying tendency of the paring force and deforming tendency of the quadrupole force in a *N*=88 nucleus can also be handled adequately .
An interesting feature of the DDM is that it often yields much better agreement with the B(E2) values and other moments than with the energies. For instance, in the case of ^{154}Gd , although the calculated energies were too high by factors of 1.4-1.7, in general reasonable agreement was obtained for the B(E2) values, magnetic moments, E2/M1 mixing ratios and E0 moments. Thus, the model can be employed for elucidating the classification of the collective bands up to quite high energies where the usual classification fails.

Now, large body of data on the B(E2) values and other moments of ^{152}Gd has been accumulated. Zolnowski *et al.,* [3,7] reported B(E2) branching ratios and X(E0/E2) ratios deduced from the decay of ^{152}Tb and from in-beam studies. Barrett *et al.,*[8] and Sharma *et al.,* [9] studies the decay of ^{152}Eu . The angular correlation work of Kalfas *et al.*, [10] and Baker *et al.,* [11] provided precise information on many $\beta \to g$ and $\gamma \to g$ transition mutipolarties. Bloch *et al.,* [12] employed the (d,d^{-})

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reaction data to established the collective nature of the low-energy states of ^{152}Gd, while Fleming *et al.,* [13] interpreted (p,t) cross section data in terms of nuclear shape changes in certain excited states. The inadequacy of nuclear theory in explaining the complex features of ^{152}Gd has been pointed out.

We have employed the IBM-2 and DDM to calculate the energy levels and transition probabilities of ^{152}Gd, and compared with experimental data.

The Models

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The Interacting Boson Model-2 (IBM-2)

In the IBM-2 the structure of the collective states in even-even nuclei is calculated by considering a system of interacting neutron (ν) and proton (π) boson s ($l=0$) and d ($l=2$). The boson Hamiltonian can be written as [14]:

$$H = \varepsilon_d (n_{d\nu} + n_{d\pi}) + \kappa Q_\nu^{(2)}.Q_\pi^{(2)} + M_{\pi\nu} \quad \text{.....(1)}$$

Where $Q_p = (s_p^{+} d_p + d_p^{+} s_p)^{(2)} + \chi_p (d^{+}{}_p d_p)^{(2)} \qquad \rho = \nu, \pi$

and $M_{\pi\nu} = \xi_2 (s_\nu^{+} d_\pi^{+} - d_\nu^{+} s_\pi^{+})^{(2)} (s_\nu d_\pi - d_\nu s_\pi)^{(2)} - 2\sum_{k=1,2} \xi_k (d_\nu^{+} d_\pi^{+})^k (d_\nu d_\pi)^{(2)}$

The Majorana term Mπν introduced the force which creates the K=1 band in deformed nuclei and push up all states with an amount that depends only on their symmetry with respect to interchange of neutron and proton:

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu \qquad \text{.....(2)}$$

Where e_π and e_υ are proton and neutron effective charges. The M1 transition operator is:

$$T(M1) = \sqrt{3/\pi}\,(g_\pi L_\pi + g_\upsilon L_\upsilon) \qquad \text{.....(3)}$$

where $L_\upsilon (L_\pi)$ is the neutron and (proton) angular momentum operator

$$L_p^{(1)} = \sqrt{10}(d^{+} d)^{(1)}$$

where $\rho = \pi, \nu,$ g_π and g_υ are the effective boson (proton, neutron) geomagnetic –factors.

Instead of evaluating the E2 and M1 matrix elements for the ^{152}Gd isotope under study which are essential in the theoretical mixing ratio calculations, it is possible to determine these ratios in an analytical form. The calculated reduced E2/M1 mixing ratio is:

$$\Delta(\mathrm{E2/M1}) = \frac{\langle \mathrm{I_f} \| \mathrm{T(E2)} \| \mathrm{I_i} \rangle}{\langle \mathrm{I_f} \| \mathrm{T(M1)} \| \mathrm{I_i} \rangle} \qquad (4)$$

related to mixing ratios, $\delta(\mathrm{E2/M1})$ by

$$\delta(\mathrm{E2/M1}) = 0.835 \mathrm{E}_\gamma \Delta(\mathrm{E2/M1}), \qquad (5)$$

where E_γ is called the transition energy measured in MeV while $\Delta(\mathrm{E2/M1})$ is measured in $(\mathrm{eb}/\mu_\mathrm{n})$.

In the IBM-2, the monopole transition (E0) operator is given by [15].

$$T(E0) = \beta_{0\rho} (d^{+} \times d)_\rho^{(0)} + \gamma_{o\rho} (s^{+} \times s)_\rho^{(0)} \qquad (6)$$

which is related to the transition matrix $\rho(E0)$ by the expression:

$$\rho_{if}(E0) = \frac{Z}{R_o^2} \sum \beta_{0\rho}^{-} < f \left| d_\rho^{+} \times d_\rho \right| i > \text{.....} \text{.....(7)}$$

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where R_0 is the nuclear radius constant (R_0=1.25*10^{-15}m).
The Monopole transition probability is defined by:

$$B(E0; J_i \rightarrow J_f) = e^2 R_0^2 \rho^2(E0) \ldots\ldots\ldots\ldots\ldots (8)$$

The branching ratio is defined by [16]

$$X(E0/E2) = 2.54\times 10^9 A^{4/3} \frac{E^5(MeV)}{\Omega_K} \alpha(E2) q^2 \ldots\ldots\ldots\ldots (9)$$

Is defined by Lange et al., [17] q^2 where

$$q^2 = \frac{(1+\delta^2)\alpha_{\exp.} - \alpha(M1)}{\alpha(E2)\delta^2} - 1$$

where δ is the multipole mixing ratio of the transition with ΔJ= 0, $J_i = J_f = 0$, and the αs denote the experimental internal conversion coefficients**.**

The Dynamic Deformation Model (DDM)

Collective bands of even-even nuclei are described in terms of the generalized Hamiltonian :

$$H = V(\beta,\gamma) + T_{vib} + T_{rot} \ldots\ldots\ldots\ldots (10)$$

where

$$T_{vib} = \frac{1}{2}(B_{\beta\beta}\beta^{.2} + 2B_{\beta\gamma}\beta\beta^{.}\gamma^{.} + B_{\gamma\gamma}\beta^2\gamma^{.2}) \ldots\ldots\ldots (11)$$

and

$$T_{rot} = \frac{1}{2}\sum_{k=1}^{3} \Im_k(\beta,\gamma)\omega_k^2 \ldots\ldots\ldots\ldots (12)$$

The scalar potential $V(\beta,\gamma)$ of equation (1), the three vibrational mass parameters of equation (11) and the three moments of inertia of equation (3) are microscopically calculated functions of the nuclear shape variables (β,γ) . No expansion is made around the potential minimum. Instead, the enharmonic terms causing rotation-vibration coupling are treated exactly.

Results and Discussion

Energy Levels

The computer program NPBOS [18] of IBM-2 was used to make the Hamiltonian diagonal. All parameters were treated as free and their values were calculated by fitting to the measured levels energies. This procedure was made by selecting the *conventional* values of the parameter and then allowing one parameter to vary while holding the others constant until the best fit was achieved. This was carried out iteratively until the overall was determined. The best fit values for the Hamiltonian parameters are given in table (1). The calculated energy levels compared with our experimental results [19] are given in table (2).

Table 1: The IBM-2 Hamiltonian parameters. All parameters are in MeV except χ_ν and χ_π, they are dimensionless.

Isotope	ε_d	κ	χ_ν	χ_π	$\xi_{1,3}$	ξ_2	$C_{L\pi}$	$C_{L\nu}$
^{152}Gd	0.720	−0.059	−1.501	−0.120	−0.090	0.120	0.0	0.0

$$C_{L\pi} = C_{L\nu} = 0.0 \quad (L=0,2,4)$$

The calculated energy levels of ^{152}Gd are compared with experiment in table (2) .

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In view of the large difference between the calculated and the experimental energies for $E > 1MeV$ in DDM, one may question whether such a comparison is meaningful. However, previous experience [20] suggests that the major deficiency of the PPQ model has comparatively smaller effects on the *K* (band) structure and on the B(E2) values. Hence, the DDM can be employed as a reasonably reliable guide for quite high even parity states.

According to the calculated energy levels in DDM given in table (2), the ground state bands are almost purely *K*=0 but the higher band members have substantial *K* mixing. In spite of this, the rotational model classification of g, β and γ bands has been employed on the basis of the major components for the lowest three bands. The $R_{4/2}$ ratios of the ^{152}Gd is 2.2 in IBM-2, and 1.96 in DDM but is 2.19 for experimental data . The 4_1^+ level is nearer to 0_2^+ state. It indicates that ^{152}Gd nucleus lie nearer to SU(5) limit of IBM-1 (vibrational shape).

Table 2: Level Energy of ^{152}Gd in (MeV Units)

Levels J_i^+	Exp. [19]	DDM	IBM-2
2_1	0.344	0.352	0.344
4_1	0.755	0.690	0.759
6_1	1.227	1.086	1.189
0_2	0.6153	0.756	0.567
2_2	0.930	1.239	0.853
4_2	1.282	1.834	1.279
2_3	1.109	1.742	1.214
3_1	1.434	1.975	1.322
4_3	1.550	2.173	1.419
5_1	1.861	2.342	1.764
0_3	1.047	2.000	0.981
2_4	1.318	2.282	1.256
2_5	1.605	2.650	1.675
3_2	1.839	3.106	1.760
4_4	1.692	2.960	1.721
4_5	-	3.004	2.341
5_2	-	3.084	2.761
2_6	1.862	3.007	1.760
2_7	1.914	3.485	2.037

Electric Transition Probability

NPBOS code has been used to calculate the transition matrix elements. Electric quadrupole transition probability B(E2) have been calculated using the effective charge $e_\pi = 0.14$ eb and $e_\upsilon = 0.250$ eb which has been estimated using the method described in reference [21] in IBM-2. The results of the calculation of the B(E2) matrix elements are shown in table (3).

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The $B(E2;2_1^+ \to 0_1^+)$ and $B(E2;4_1^+ \to 2_1^+)$ values do increase as neutron number increases toward the middle of the shell in Gd nuclei as the value of $B(E2;2_2^+ \to 2_1^+)$ has a small value as it contains admixture of M1. As a consequence of possible M1 admixture, this quantity is rather difficult to measure. The value of $B(E2;2_2^+ \to 0_1^+)$ is small because this transition is from quasi-beta band to ground state band (cross over transition). The IBM-2 calculations for B(E2) is closer to experimental data than DDM calculations.

Comparison with the experimental B(E2) branching ratios is presented in table 4 for transitions from β band. In general, the present IBM-2 and DDM results are considerably closer to experiment. One possible reason is that a larger configuration space is employed. Another reason is that a perturbation expansion around the spherical shape is avoided. In the second case case, the B(E2) ratios of the IBM-2 are closer experiment.

The results of table 4 again provide justification for grouping the states of even a spherical nucleus like ^{152}Gd into collective bands. For instance, note the large branching ratios for $(2_2^+ \to 0_2^+)/(2_1^+ \to 0_1^+)$ in table 4.

Table 3: Electric Transition Probability B(E2) (e^2b^2) of ^{152}Gd

Transition	Exp. [19,22]	DDM	IBM-2
$2_1^+ \to 0_1^+$	0.773(15)	1.63	0.772
$4_1^+ \to 2_1^+$	1.178(2)	0.045	1.093
$6_1^+ \to 4_1^+$	1.388(12)	0.057	1.243
$2_2^+ \to 2_1^+$	0.0600(9)	0.14	0.021
$2_2^+ \to 0_1^+$	$4*10^{-3}(10)$	1.21	0.0034
$2_2^+ \to 0_2^+$	0.49(16)	2.3	0.521
$4_2^+ \to 2_2^+$	$3*10^{-3}$	0.004	0.0027

Table 4: B(E2) Branching ratios of ^{152}Gd

Transition	Exp. [3]	DDM	IBM-2
$2_2^+ \to 0_1^+ / 2_2^+ \to 2_1^+$	0.020 (3)	0.097	0.031
$2_2^+ \to 4_1^+ / 2_2^+ \to 2_1^+$	2.04(27)	1.61	1.27
$2_2^+ \to 0_2^+ / 2_2^+ \to 0_1^+$	107(11)	32	111
$2_2^+ \to 0_2^+ / 2_2^+ \to 2_1^+$	2.1(4)	2.96	2. 17 (4)
$4_1^+ \to 2_1^+ / 4_2^+ \to 4_1^+$	-	0.01	0.054
$4_1^+ \to 6_1^+ / 4_2^+ \to 4_1^+$	-	1.26	0.06

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$4_1^+ \to 2_2^+ / 4_2^+ \to 4_1^+$	8.4 (8)	8.1	7.6
$6_1^+ \to 4_2^+ / 6_2^+ \to 6_1^+$	16(6)	13.9	18

$\delta(E2/M1)$ Mixing Ratios

The calculated $\delta(E2/M1)$ mixing ratios are compared with experiment in table 5. The agreement is quite poor compared with the experimental data. The most probable reason is that the M1 transition matrix elements, which are reduced by one or more orders of magnitude compared with the static matrix elements are much more sensitive to the details of the dynamic and, hence, to the effects of configuration space truncation as compared with the E2 matrix elements.

Table 5: $\delta(E2/M1)$ mixing ratio for ^{152}Gd

Transition	Exp. [17]	DDM	IBM-2
$2_2^+ \to 2_1^+$	-3.0 (3)	27.4	-2.89
$2_3^+ \to 2_1^+$	$4.3^{+0.7}_{-0.6}$	36.4	10
$2_4^+ \to 2_1^+$	0.58(7)	-3.4	1.21
$2_5^+ \to 2_1^+$	-	0.64	0.78
$2_7^+ \to 2_4^+$	$0.7^{+0.8}_{-0.5}$	-20	0.23
$2_7^+ \to 2_5^+$	-	-11	-16
$3_1^+ \to 2_1^+$	-0.29 (5)	84	-0.32
$3_1^+ \to 4_1^+$	24	34	20
$4_2^+ \to 4_1^+$	-	43	21

Electric Monopole transition Probability

The calculated electric monopole moments $\rho(E0)$, and the mixing ratio $X(E0/E2)$ values are compared with experiment in table 6. Note that the sign of the transition moments $\rho(E0)$ depends on the relative phase of the final wave functions and is, hence, phase-convention dependent.

As pointed out previously [23], a large $X(E0/E2)$ value is not necessarily a signature of a β-vibrational state, for instance, our calculated $X(E0/E2)$ value for $2_2^+ \to 2_1^+$ transition. However, it should be kept in mind that large results were obtained from the vanishing B(E2) values, especially in the case of higher bands whose structure may be quite different from that of the lower bands. Because of the possibility of accidental cancellations in the calculation of a sum of terms with different signs, only the correct order of magnitude can be expected from present calculation of a large number of states and matrix elements.

Table 6: Electric monopole moments $\rho(E0)$ and $X(E0/E2)$ for ^{152}Gd

Transition		$\rho(E0)$			$X(E0/E2)$		
J_i^+	J_f^+	Exp. [17]	IBM-2	DDM	Exp. [17]	DDM	IBM-2
0_2^+	0_1^+	10(36)	12	25	1.45 (11)	1.46	6.0
2_2^+	2_1^+	-	-20	-24	6.9 (16)	10.6	52.8

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4_2^+	4_1^+	-	13	24	23.4 (23)	16.5	86.5
0_3^+	0_1^+	-	2.01	3	-	11	-
0_3^+	0_2^+	-	10	31	-	9	3.1
2_4^+	2_1^+	-	3.45	5.4	9 (1)	22	-
2_4^+	2_2^+	-	18	26	31 (8)	12	-
2_5^+	2_1^+	-	6.98	7.3	-	430	-
2_5^+	2_2^+		4	5.6	-	4.2	-

Conclusion

From the present work of the ^{152}Gd, we conclude that the DDM is capable of elucidating the nuclear structure of the *N=88* isotones as successfully as for the more deformed $N \geq 90$ nuclei like ^{152}Sm and ^{154}Gd [24,25]. The delicate balance between the residual quadrupole and pairing force observed in ^{150}Sm and ^{148}Nd is not quite so critical in ^{152}Gd because of reasons discussed above. In fact, the good results, for the B(E2) values for the more favourable case in ^{152}Gd in our preliminary studies [20,26] paved the way for a clear understanding of the more complex structure of ^{150}Sm. Thus, the previous impression of a breakdown of the DDM for the B(E2) values of ^{150}Sm is now changed completely from these studies .

The IBM-2 that give considerable data were in better agreement with the experimental data of ^{152}Gd than the DDM model. We conclude that the direct solution of Schrodinger equation is far superior to a perturative around spherical shape expansion. The version of DDM allows not only for an exact solution of the Schrodinger equation but also practical calculations in a configuration space which is considerably larger than that employed in many of the currently employed models: the IBM-2 and the conventional shell model.

The present calculation shows that the energy spectrum of the even-even ^{152}Gd nucleus can be reproduced well, including a large number of energy levels. The IBM-2 yields a sequence of the ground , beta and gamma bands and multi-phonon $K^\pi = 0_3^+, 2_2^+$ bands.

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